Democratic and PopularRepublic of Algeria
Ministry of Higher Education and ScientificResearch
University of Tissemsilt

Faculty of Science and Technology

Department of Material Sciences

End-of-studies thesis for the attainment of a Master's degree in academic studies in

Field :Material Sciences

Specialization : nano physique

Presentedby :

Chouiheb Bilal & Gharbi Bilal

Theme

Study of the Impact of Nanofluid on a Stretching Cylinder

Defended on, ……………………….

In front of the Jury:

DahmaneFethallah	Examiner	Prof.	Univ-Tissemsilt
Boumia Lakhdar	Supervisor	M.C.A.	Univ-Tissemsilt
Mohamed Boucharef	President	M.C.B.	Univ-Tissemsilt

Academicyear: 2022-2023

Gratitude :

I wouldlike to thankProfessorDahmaneFethallah for the honorhebestowedupon me by accepting to preside over this Jury. Mohamed Boucharef.

I want to thankeveryonewho has contributeddirectly or indirectly to the completion of thiswork.

Dedications:

To mylovingfamily and cherishedfriends,

As I stand on the threshold of a new chapter in my life, I want to express myheartfelt gratitude to each and every one of you. Myjourneythroughuniversitywouldn't have been the samewithoutyourunwavering support, encouragement, and love. Family, yourbelief in me has been mygreatest source of strength. Your sacrifices, bothbig and small, have fueledmydetermination to reachthismilestone. I dedicatethisachievement to you, knowingthatyour love has been myguiding light. To mydearfriends, you've been more thancompanions; you've been my second family. Yourlaughter, camaraderie, and late-night study sessions have made thisjourneymemorable. Yourfriendshipis a treasure I will carry with me always. As I graduate, I carry your love, support, and friendshipwith me into the future. This is not justmyachievement; it's ours. Thankyou for being a part of mylife'sgreatestadventure.

Withheartfeltappreciation and love,

Table of contents:

Chapter I: Generalities about nanofluids:

Chapiter II: heat transfer

Chapter III:Mathematical Formulation of the Problem and Numerical Resolution

Introduction :

Engineers and scientists have been significantly interested in reducing energy consumption for a hundred years. Referred to as energy saving, energy conservation, or energy efficiency, various endeavors have resulted in the development of more advanced equipment. When it comes to heat exchange equipment, additional factors like space restrictions and economic benefits have further propelled this notion. Initially, the emphasis was on enhancing the heat transfer coefficient through improvements in thermal properties. This approach enabled the resolution of crucial issues such as minimizing surface temperatures and reducing the bulk and dimensions of heat exchangers.

Heat transfer is a crucial factor in diverse industrial operations. The effectiveness of these processes is typically restricted by the thermophysical characteristics of the fluids employed. The ability to efficiently dissipate the thermal energy generated within these devices is a prerequisite for reducing their size. The notable advancements in recent studies on nanofluids stem from the possibility of substantially enhancing heat transfer by introducing a small quantity of nanoparticles into a pure fluid, particularly in specific applications.

It is widely acknowledged that one of the primary challenges in miniaturizing microelectronic devices and computer processors is the efficient elimination of dissipated energy, which leads to an elevation in device temperature. As electronic components continue to shrink in size, a greater amount of heat is generated per unit surface and volume of processors, necessitating effective heat dissipation to prevent electronic materials from overheating and losing their semiconductor properties. Typical electronic components start to malfunction above 130 °C. Traditional cooling mediums like air, water, and similar fluids are insufficient in effectively dissipating heat from small electronic devices without surpassing this critical temperature threshold.

Well-designed nano-fluid systems hold promise as highly efficient heat transfer mediums for extracting heat from the next generation of computer components. Moreover, nano-fluids can offer benefits in various industrial processes beyond electronics. These processes include gas absorption using liquid carriers, chemical reactions, electricity production through combustion, cooling of internal combustion engines, high-energy weapons, laser systems, boiling in microgravity environments, nuclear reactor cooling, and applications in the field of biomedicine.

The utilization of wavy geometries is widespread in numerous engineering systems to enhance transport performance. Consequently, understanding the flow and heat transfer characteristics across wavy surfaces becomes crucial in this context. Various applications, ranging from small-scale to large-scale, rely on wavy surfaces for efficient heat transfer. These applications include solar collectors, refrigerator condensers, cavity wall insulation systems, grain storage containers, and industrial heat radiators, among others. Over the years, there has been a growing interest among researchers in applied mathematics, mechanical and chemical engineering, biomechanics, and fluid mechanics, resulting in increased focus on the study of flow and heat transfer from wavy surfaces within complex enclosures like square, trapezoidal, and rectangular spaces.

Numerous technical papers have been published on this subject, though they are dispersed across different journals. These papers cover a wide range of topics, encompassing various flow scenarios and the application of diverse mathematical techniques for analyzing complex flow situations involving wavy surfaces.

In the current work, we are studying the behavior and impact of nanofluid on a stretching cylinder. This work is organized into four chapters.

First, we present some concepts about nanofluids.

Second chapter we talking about heat transfer and their modes and equation.

The third chapter is dedicated to the mathematical formulation of the problem. The conservation equation of energy in dimensional and dimensionless form, as well as the boundary conditions, are presented.

ChapterI: Generalities about nanofluids.

I-1- Definition:

The conventional method of enhancing heat transfer in thermal systems involves increasing the surface area of cooling devices and flow velocity, or incorporating solid particles into heat transfer fluids. However, as industries like electronics, photonics, transportation, and energy supply demand more efficient heat transfer fluids, there is a need for a fresh approach to improve heat transfer and address cooling challenges. This is where the role and significance of nanofluids in heat transfer come into focus.

A substantial portion of recent research efforts has been dedicated to studying the properties of nanoparticles and nanofluids. These efforts were triggered by early experimental findings, which indicated that the addition of a small fraction (typically less than 1%) of specific types of nanoparticles to a base fluid such as water, oil, or ethylene glycol could significantly enhance the thermal conductivity of the mixture.[1]

Nanofluids are created by dispersing metal or metallic oxide nanoparticles into carefully chosen base fluids. As nanofluids consist of two distinct phases, solid and liquid, their thermophysical properties differ significantly from those of the individual phases. The beneficial properties exhibited by nanofluids have sparked the interest of numerous researchers, leading to extensive studies conducted in various environments. These studies encompass industrial engineering processes, oil and gas industries, automobile manufacturing processes, nuclear engineering models, chemical engineering processes, renewable energy applications such as solar collectors, and more.

While there are several existing books on nanofluids, some focus primarily on numerical studies, while others emphasize experimental research. This book aims to provide a comprehensive resource that encompasses mathematical, numerical, and experimental studies in the field of nanofluids. Its purpose is to bridge the existing gap in available texts and offer a comprehensive understanding of the subject matter. [2]

The following nanofluid behavior has been observed consistently by different researchers at different organizations and with different nanofluids:

- Abnormal thermal conductivity increase relative to the base fluid. Eastman et al.[3] reported a 40% increase in the thermal conductivity of ethylene-glycol with 0.3 vol % copper nanoparticles of 10 nm diameter. Das et al.[4] have observed increases of 10–25% in water with 1–4 vol % alumina nanoparticles. Also, it appears that thermal conductivity of nanofluids is a strongly increasing function of temperature, much more so than that of pure liquids.

Abnormal viscosity increase relative to the base fluid. Pak and Cho [5] measured the viscosity of alumina/water and titania/water nanofluids at 1–10 vol %, and found it to be much higher than that of pure water, well beyond the prediction of traditional viscosity models such as BrikmanEinstein’s [6] or Batchelor’s[7]. The same conclusion was reached by Maïga et al[8]. while correlating the viscosity data of Lee et al[9]. and Wang et al[10].

Abnormal single-phase convective heat-transfer coefficient increase relative to the base fluid. Pak and Cho [5] reported heat transfer data for turbulent flow of alumina/water and titania/water nanofluids in circular tubes. Their data show Nusselt numbers up to about 30% higher than predicted by the pure fluid correlation (Dittus-Boelter), even though the measured nanofluid properties were used in defining the dimensionless groups in the correlation.Nusselt numbers over 30% higher than the Dittus-Boelter correlation were also reported by Xuan and Li [11] for turbulent flow of copper/water nanofluids.

Since Choi conceived the novel concept of nanofluids in the spring of 1993, talented and studious thermal scientists and engineers in the rapidly growing nanofluids community have made scientific breakthrough not only in discovering unexpected thermal properties of nanofluids, but also in proposing new mechanisms behind enhanced thermal properties of nanofluids, developing unconventional models of nanofluids, and identifying unusual opportunities to develop next-generation coolants such as smart coolants for computers and safe coolants for nuclear reactors. As a result, the research topic of nanofluids has been receiving increased attention worldwide. The recent growth of work in this rapidly emerging area of nanofluids is most evident from the exponentially increasing number of publications. [12]

I-2- Characteristics of nanofluids.

Nanofluids have shown significant potential among many applications. They are composed of suspensions containing nanometer-sized particles, tubes or fibers. Metallic, carbon-based, ceramic, or nonmetallic particles can be employed. Moreover, hybrid nanofluids made from a mixture of different suspensions also reveal interesting flow behavior. For instance, a binary mixture of iron and copper oxide revealed distinct Newtonian and non-Newtonian behavior at low and high concentrations, respectively .The common base fluid includes water, oil, or sometimes ethylene glycol (EG). Many researchers have reported improvement in thermal conductivity with very small concentrations (normally less than 1% volume fraction of nanoparticles), especially in microchannel application. Such improvements are related to the particle characteristics, size and shape, volume concentration, and the type of base fluid. Researchers are now trying to stabilize the nanopar- ticle suspensions for prolonged periods, increase the thermal conductivity, reduce the pressure drop, and reduce erosion problems in MCHS. Therefore, before using nanofluids in MCHS, the knowledge of their thermophysical properties is essential for characterizing performance. In this section, the authors will discuss the basic properties with examples.[13]

I-3- Various Nanofluids and Their Attributes :

system	Synthesisprocess	Particleloading(vol.%)	Particle size (nm)	Enhancement in thermal conductivity(%)
Cu/EG	$Single-step$	0.3	10	40
Cu/H_2O	$Single-step$	0.1	$75-100$	23.8
$Cu//H_2O$	$Two-step$	7.5	100	78
Fe/EG	$Single-step$	0.55	10	18
$Ag90/toluene$	$Two-step$	0.001	$60-80$	16.5
$Au/toluene$	$Two-step$	0.0026	$10-20$	21
$Au/ethanol$	$two-step$	0.6	4	1.3 ∓ 0.8
Fe_3O_4/H_2O	$single-step$	4	10	38
TiO_2/H_2O	$Two-step$	5	15	$30-33$
Al_2O_3/EG	$Two-step$	0.05	60	39
CuO/H_2O	$Two-step$	5	33	11.5
Sic/H_2O	$Two-step$	4.2	25	15.9
$NCTs/engine\ oil$	$Two-step$	2	$20-50$	30
$NCTs/poly\ oil$	$Two-step$	1	$25nm \times 50\mu m$	160
$NCTs/EG$	$Two-step$	1	$15 \times 30\mu m$	19.6
$NCTs/H_2O$	$Two-step$	1	$15 \times 30\mu m$	7
$NCTs/decene$	$Two-step$	1	$15 \times 30\mu m$	12.7
$H_2O/FC-72$	$Two-step$	12	$9.8nm$	52
Al_2O_3/H_2O	$Two-step$	5	20	20

Table1 Li, Y. et al.,*Powder Technol*ogy, 196, 89–101, 2009

I-4- Preparation Methods for Nanofluids :

Two different techniques have been applied to produce nanofluids. One is a single-step method, and the other is a two-step method. Further, there are two ways to produce hybrid nanofluids: (a) addition of two different types of nanoparticles into base fluid and (b) fabrication of nanocomposites and dispersion into base fluid. The former method is less popular among the researchers, although the preparation process is very simple. By simply adding two different types of nanoparticles into a base fluid, one may not able to fully utilize improved synergistic physicochemical properties of the individual materials. In the latter method, fabrication of nanocomposites is a tedious and complex process. The two different nanoparticles that are bound together must be stable enough to sustain its dispersion into base fluid. If they break apart during dispersion, they will become similar to adding two different nanoparticles into base fluid.[14]

I-4-1- One-Step Method

The one-step method is a process wherein nanoparticle fabrication is combined with nanofluid synthesis. Several single-step methods have been arrived for nanofluids preparation.

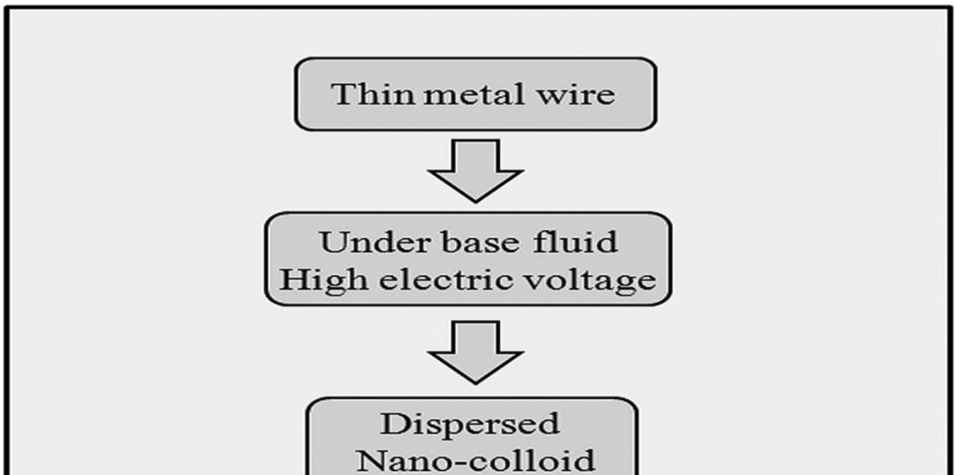
I-4-1-1- Vapor Deposition:

Direct evaporation and condensation of the nanoparticulate materials in the base liquid are done to produce stable nanofluids. A more complex method is evaporating bulk metal precursors which can be in the form of wires in an inert-gas atmosphere, which is then followed by vapor condensation. The size of nanoparticles depends on degree of heat applied for evaporation, as with increase in super heat, the particle size reduces. The particle size decreases with increase in pressure of the inert gas and also with decrease in wire diameter. During vapor condensation, nanoparticle sizes and the particle size distribution are controlled by regulating the condensation conditions. Other techniques that use metal evaporation include cathodic spraying or sputtering, plasma arc, laser ablation, lithography, radiolysis, and resistive heating.

The nanoparticles can also be produced by their condensation from the vapor phase into a flowing low vapor pressure fluid such as oil which is called “vacuum evaporation onto a running oil substrate” . In this method, drying, storage, transportation, and dispersion of nanoparticles are avoided, so the agglomeration of nanoparticles is minimized and the stability of the nanofluids is increased . A disadvantage of this method is that it is impossible to scale it up for industrial applications and is applicable only for low vapor pressure fluids. Another approach is chemical vapor deposition, wherein vapor phase precursors are brought into a hot-wall reactor under conditions that favor nucleation of particles in the vapor phase rather than deposition of a film on the wall. M. A. Akhavan used chemical vapor deposition process to produce multiwalled carbon nanotubes (MWCNTs) nanofluids with deionized water.

I-4-1-2- Electrical Explosion of Wire:

A promising one-step method is the electrical explosion of wire (EEW) in the desired base fluid. EEW technique uses high electric voltage and current to cause explosion of a thin metal wire in a container of base fluid to produce the nanofluid (Figure 1.1). Any type of thin conductive wire can be transformed into nanoparticles by this method. SadeghAberoumand and Amin Jafarimoghaddam prepared a colloidal suspension consisting of Cu NPs and engine oil by EEW in three different weight concentrations of 0.2%, 0.5%, and 1% .(2) A novel approach to EEW is µ-micro-electrical discharge machining technique wherein material is removed from both the electrodes through melting and evaporation. Sahu et al. synthesized copper nanofluids by µ-micro-electrical discharge machining technique . Lo et al. introduced a novel system to prepare nanofluid using submerged arc nanoparticle synthesis system technique in which a pure metal rod is heated by a submerged arc formed by electrodes as depicted in Figure 1.



- ***Figure1*** *Schematic of the EEW method.*

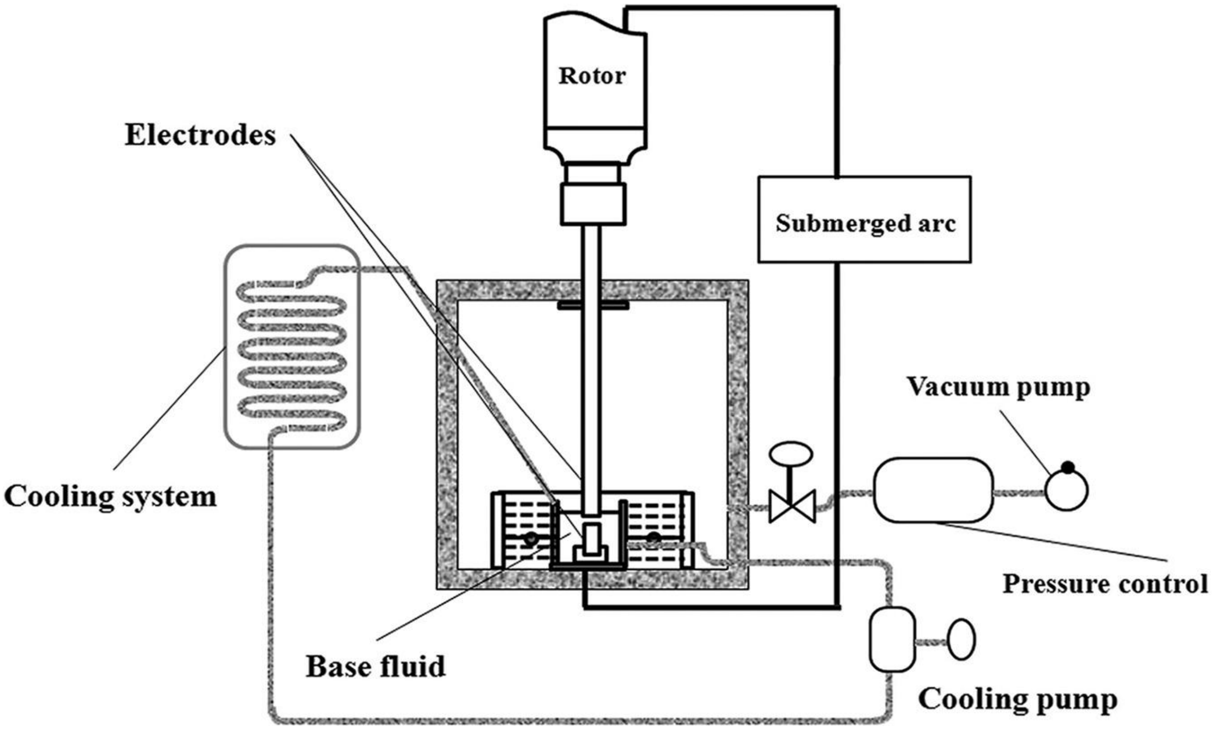


Figure2 *Schematic of the EEW method.*

I-4-2- Two-Step Method:

The main advantage of one-step method is low agglomeration of NPs, which leads to improved stability of nanofluids. Cost of drying and dispersion can be avoided using this method. However, there are some disadvantages for one-step method. Major disadvantage is that it is impossible to scale it up due to high cost of production, and it is compatible with only low vapor pressure base fluids . Nine et al. reported the preparation of Cu2O-water hybrid nanofluid by using wet ball milling process . The ball milling accelerated hydrolysis of Cu particles in the presence of de-ionized water and several Cu particles transformed to Cu2O nanoparticles at a temperature slightly higher than ambient temperature.[14]

I-5- Proprieties :

I-5-1- Thermal Conductivity of Nanofluids:

One of the most significant thermophysical properties to study the capability of nanofluids in heat transfer area is its thermal conductivity. For effective nanofluids, higher thermal conductivity is needed for heat transfer applications. Thermal conductivity influences Nusselt and Prandtl numbers, which are required for heat transfer characteristics of the fluid.

There are various models involved in the calculation of thermal conductivity and the models are listed below:

I-5-1-1- Maxwell Model:

Maxwell model is based on effective medium theory, and this model can be used for randomly dispersed and uniformly sized spherical particles .

$$\frac{k_{eff}}{k_f}=\frac{k_p+2k_f+2\emptyset(k_p-k_f)}{k_p+2k_f-\emptyset(k_p-k_f)}$$

where, k_{eff} is the effective thermal conductivity of nanofluid, k_f is the thermal conductivity of base fluid,k_p is thermal conductivity of nanoparticle, and ϕ is the volume fraction of nanoparticle.

I-5-1-2- Hamilton–Crosser Model:

Hamilton–Crosser model is applicable for spherical and cylindrical particles, and it was developed by using shape factor.

$$\frac{k_{eff}}{k_f}=\frac{k_p+(n-1)-(n-1)\emptyset(k_f-k_p)}{k_p+(n-1)k_f+\emptyset(k_f-k_p)}$$

where$n$is the shape factor and n= 3 for spheres and n= 6 for cylinder. Figure 3 shows the comparison of experimental and calculated thermal conductivity values using Hamilton–Crosser model for hybrid nanofluids.

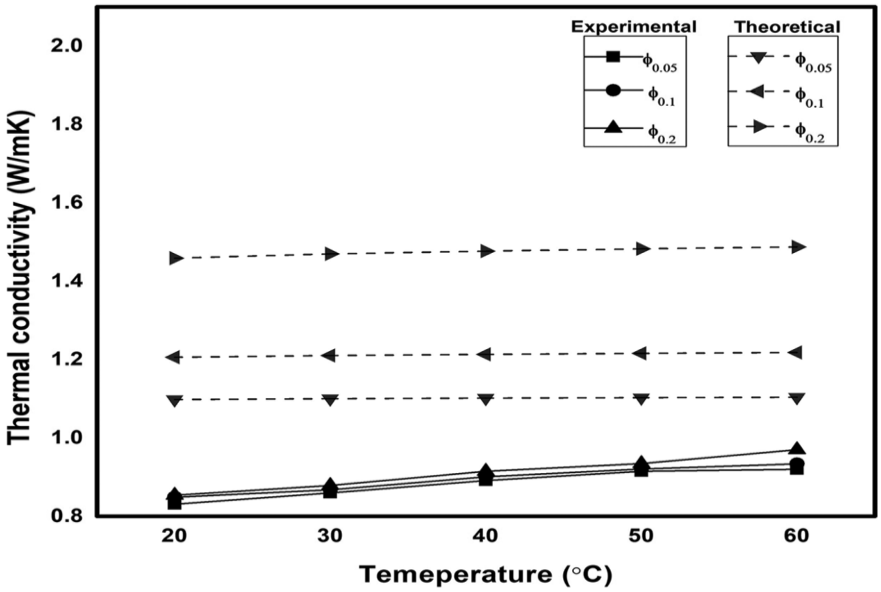


***Figure3** Comparison of experimental and theoretical thermal conductivity for hybrid nanofluid. (From Kannaiyan, S.et al., J. Mol. Liq., 244, 469–477, 2017.)*

I-5-1-3- BruggemanModel :

Bruggeman model is applicable for binary mixtures of homogeneous randomly dispersed and spherical nanoparticles.

$$\frac{k_{eff}}{k_f} = \frac{1}{4}\left[(3\emptyset - 1)\frac{k_p}{L_f} + (2 - 3\emptyset) + \frac{k_f}{4}\sqrt{\Delta}\right]$$

where, Δ is the thermal conductivity of suspension regardless of particle motion and vibration.

I-5-1-4- Factors affectingthermal conductivity :

There are various parameters, which can control thermal conductivity of nanofluids, and we count them below:

-Effect of pH and Surface Modifier.

-Effect of Temperature and Base Liquid.

-Effect of Nanoparticle Size, Shape, and Concentration.

-Effect of Fabrication Method.

I-5-2- Viscosity of Nanofluids:

For cooling applications, it is not only sufficient to achieve high thermal conductivity, but also viscosity should be assessed. Viscosity is defined as the internal resistance to flow, and it is a valuable property in all the thermal applications. Viscosity increases by adding nanoparticles to the base fluid, and this often causes an adverse effect on pumping power and heat transfer coefficient. Viscosity is influenced by several factors, and they will be discussed in the following sections.

I-5-2-1- Einstein Model:

The nanofluid viscosity formula was first developed by Einstein in 1906, and the model was based on the assumption that the viscous fluid contains spherical particles at a very low volume fraction (ϕ< 0.02). The Einstein model shows a linear increase in viscosity with particle volume concentration, and it has the limitation, since it does not the structure and particle concentration . Further, this model cannot be applied to high particle concentration. The Einstein model is given below:

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5\emptyset$$

where, $\mu_{nf}$is the viscosity of the nanofluid, $\mu_f$is the viscosity of the base fluid, and $\phi$is the volume fraction of the particle in suspension. Figure 4 represents the comparison of experimental and theoretical viscosity (calculated using Einstein model) for hybrid nanofluids.

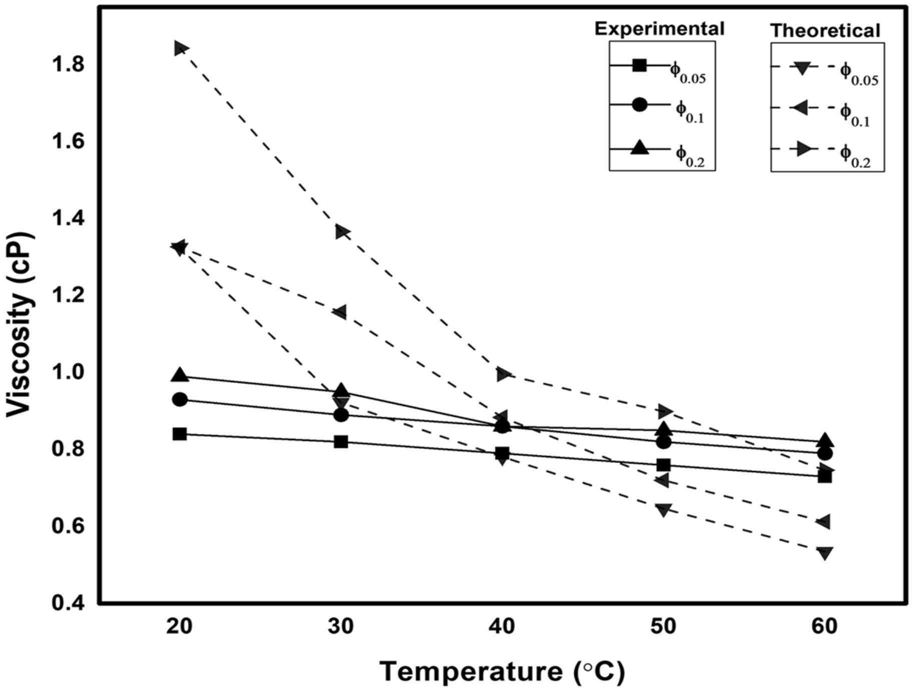


Figure4*Comparison of experimental and theoretical viscosity for hybrid nanofluid. (From Kannaiyan, S. et al., J. Mol. Liq., 244, 469–477, 2017.)*

I-5-2-2- Krieger and Dougherty Model:

For randomly dispersed spherical particles, a semi empirical model was proposed by Krieger and Dougherty in 1959 . The model is stated below: $\frac{\mu_{nf}}{\mu_f} = \left[1 - \frac{\emptyset}{\emptyset_m}\right]^{-\eta\emptyset_m}$

where$\emptyset_m$is the maximum particle packing fraction, which varies from 0.495 to 0.54 and is approximately 0.605 at higher shear rates, and $\eta$is the intrinsic viscosity whose value is 2.5 for monodispersedsuspensions of hard spheres.

I-5-2-3- Mooney Model :

For higher concentrations of spherical suspension, Mooney proposed a model in 1951, and the model is given by the below expression.

$$\frac{\mu_{nf}}{\mu_f}=e^{\left(\frac{\xi\phi}{1-k\phi}\right)}$$

where$k$ is a constant, called self-crowding factor ($1.35 < k < 1.91$) and $\xi$is called the fitting parameter whose value is 2.5

I-5-2-4- Factors affecting Viscosity:

Viscosity is influenced by several factors, and they will be discussed in the following sections:

-Surface Modifiers

-Nanoparticles Size, Shape, and Concentration.

-Temperature, pH, and Preparation Method.

I-5-3- Density of Nanofluids :

The pressure drop and pumping power in engineering applications also depend upon the density of nanofluids.

The density of nanofluids is the least discussed property yet it has a significant effect on heat transfer performance and has been observed to undergo positive change when temperature and nanoparticle concentration are increased.[13]

I-5-3-1- Factors affectingDensity:

The density of nanofluids depends on several factors mentioned below:

-Temperature, Concentration, and Surface Modifiers.

I-5-4- Specific Heat of Nanofluids :

The effect of nanoparticle addition on the specific heat of ionic liquids is quite inconclusive as some of the researchers have reported a positive impact of the addition of nanoparticles on the specific heat, while others have reported either no influence or a negative effect. Similarly, the temperature has been reported to have the same effect as the nanoparticle concentration has on the specific heat of the nanofluids.[13]

I-5-4-1- Models for Specific Heat

Two models are widely used in the literature for calculating the specific heat, and it is simple to calculate specific heat. Model I is similar to mixing theory for ideal gas mixtures, which is a straight average relating specific heat of nanofluid, $c_{p,nf}$, to the specific heat of base fluid, $c_{p,f}$, specific heat of nanoparticles, $c_{p,n}$, and volume fraction, ϕ[66]. The nanofluid specific heat using Model I is given as follows:

$$c_{p,nf} = \varphi c_{p,n} + (1-\varphi)c_{p,f}$$

Model Ⅱ was proposed based on the assumption of thermal equilibrium between the particles and the surrounding fluid. Model Ⅱ considers the effect of particle and fluid densities (ρ_n and ρ_f, respectively) on specific heat and the Model Ⅱ is given below:

$$c_{p,nf} = \frac{\varphi(\rho c_p)_n + (1-\varphi)(\rho c_p)_f}{\varphi\rho_n + (1-\varphi)\rho_f}$$

I-5-4-2- Factors affecting Specific Heat:

-Temperature and Volume Fraction.

-Nanoparticle Size and Shape .

-Influence of Surfactant .

I-6- Applications:

Much of the work in the field of nanofluids is being done in national laboratories and academia and is at a stage beyond discovery– research. However, a great number of companies in the United States and other countries have been showing great interest and suggesting a number of possible applications of nanofluids. This great industrial interest shows that nanofluids can be used for a wide variety of industries, ranging from electronics, transportation, HVAC, and process heating and cooling to energy conversion and supply and magnet cooling.[15]

There are many fields of nanotechnology. In this study, we'll be focusing on the nanofluids and heat transfer field:

I-6-1- Heat Transfer Application :

Nanofluids have received attention as potential heat transfer fluids with enhanced thermal properties and heat transfer performance. Because of these characteristics, there are strong possibilities for improving the efficiency of existing heat transfer processes. These processes can be applied industrial cooling, smart fluids, the heating of buildings, the extraction of geothermal power and other energy sources, heat exchangers, nuclear reactors, and space and defense. We reviewed industrial cooling, heat exchangers, and carbon nanotube (CNT) nanofluids.

I-6-1-1- Industrial Cooling :

The main application of nanofluids is with heat transfer fluids in closed-loop liquid cooling systems. These systems are utilized in many applications including chemical/rubber processing, the semiconductor industry, petroleum refineries, electrical systems, computing, power generation, power electronics, production machinery, and combustion engines[16]. Ma et al. [17]suggested for the first time the concept of a nano liquid-metal fluid, aiming to establish an engineering route to make a highest conductive coolant. Using several widely accepted theoretical models for characterizing a nanofluid, the thermal conductivity enhancement of a liquid metal fluid due to addition of more conductive nano-particles was predicted by Ma et al. Figure 5 shows a tube radiator and its thermal resistance.

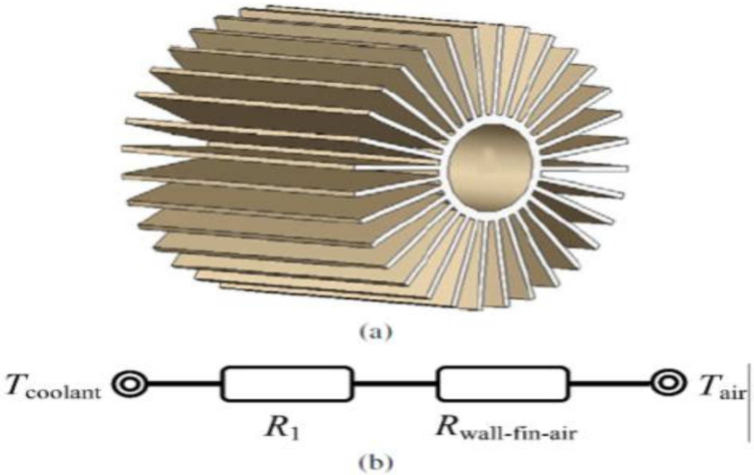


***Figure5** Heat transfer using flowing liquid running inside a tube radiator. (a) Cylindrical tube radiatorwith fin structure. (b) Thermal resistance betweencoolant and ambient air.*

I-6-1-2- Heat Exchangers :

Heat exchangers are widely utilized in engineering applications including the chemical industry, power production, food industry, environmental engineering, waste heat recovery, air conditioning, and refrigeration[18].

Chun et al[19]. studied the convective heat transfer coefficient of nanofluids made of several types of alumina nanoparticles and transformer oil flowing through a double pipe heat exchanger system in the laminar flow regime. The nanofluids exhibited a considerable increase in their heat transfer coefficient. Although the thermal conductivity of alumina is not high, it is much higher than that of the base fluids. Khoddamrezaee et al[20]. investigated the characteristics of (EG+Al2O3) nanofluid and (EG) fluid crossing a rectangular arrangement of tubes in a shell-and-tube heat exchanger. Figure 6 shows velocity contours of a nanofluid. Results show that by using a nanofluid, the stagnation and separation points of the flow were postponed, and the heat transfer coefficient and shear stress increased.

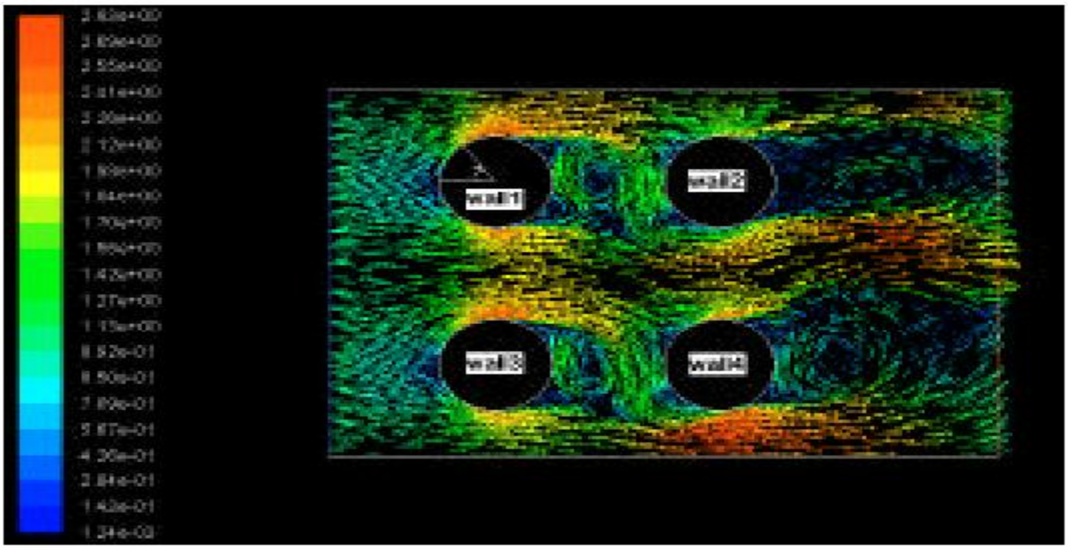
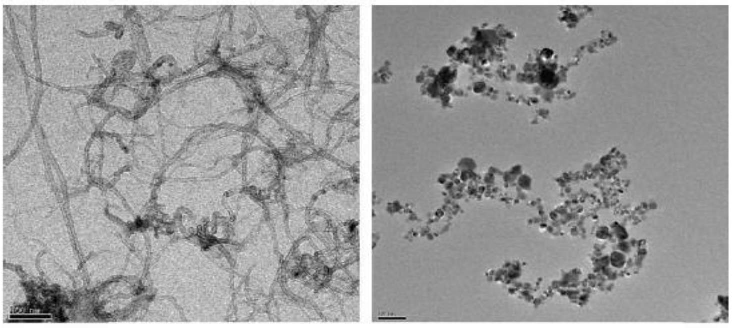


Figure6 *Velocity contours of nanofluid in a heat exchanger.*

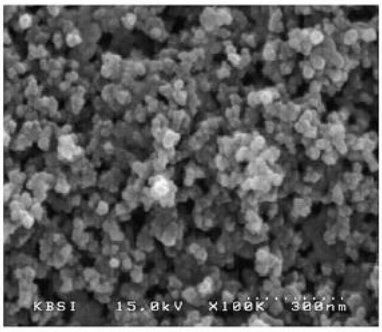
I-6-1-3- CNT Nanofluids:

The field of nanofluids was highly developed along with development of nanotechnology. Also, carbon nanotubes came to the fore because of their outstanding thermal properties. Many studies are underway in the field of CNT nanofluids.

Hwang et al[21]. produced four kinds of nanofluids: multiwalled carbon nanotubes (MWCNTs) in water, CuO in water, SiO2 in water, and CuO in ethylene glycol. Their thermal conductivities were measured using the transient hot wire method. The results show that the thermal conductivity enhancement of nanofluids depends on the thermal conductivities of the particles and the base fluid. Figure 7 shows photographs of test particles. Han et al[22]. used phase change materials as nanoparticles in nanofluids to simultaneously enhance the effective thermal conductivity and specific heat of the fluids.



(a) MWCNT (b) CuO nanoparticle



(c) SiO_2 nanoparticle

Figure7 *Photographs of test particles.*

I-6-2- Cooling Applications:

I-6-2-1- Crystal Silicon Mirror Cooling:

One of the first applications of research in the field of nanofluids is for developing an advanced cooling technology to cool crystal silicon mirrors used in high-intensity x-ray sources (Lee and Choi, 1996). Because an x-ray beam creates tremendous heat as it bounces off a mirror, cooling rates of 2000 to 3000 W/cm2 should be achievable with the advanced cooling technology. Lee and Choi carried out analysis to estimate the performance of microchannel heat exchangers with water, liquid nitrogen, and nanofluids as the working fluid. For an optimized channel width that minimizes the thermal resistance of a microchannel heat exchanger, performance of a nanofluid-cooled microchannel heat exchanger has been compared with that of water-cooled and liquid-nitrogen-cooled microchannel heat exchangers. The results show that nanofluids can remarkably reduce the thermal resistances and increase the power densities, so they demonstrated the superiority of a nanofluid-cooled silicon microchannel heat exchanger. The benefits of using nanofluids as a room-temperature coolant are clear, including dramatic enhancement of cooling rates while operating the advanced cooling system at room temperature. Furthermore, the possibility of thermal distortion and flow-induced vibration will be eliminated by passing the nanofluids through microchannels within the silicon mirror itself.[12]

I-6-2-2- Cooling of microchips:

Nanofluids are used for the cooling of microchips in computers. The enhanced characteristics of nanofluids can increase the efficiency of microchips and facilitate their development. Ma et al[23].developed an ultrahigh-performance cooling device called the "nanofluid oscillating heat pipe" by combining nanofluids with thermally excited oscillating motion in an oscillating heat pipe (OHP). Experimental results show that when the OHP is charged with nanofluid, the heat transport capability significantly increases. Figure 8 shows a nanofluid OHP.

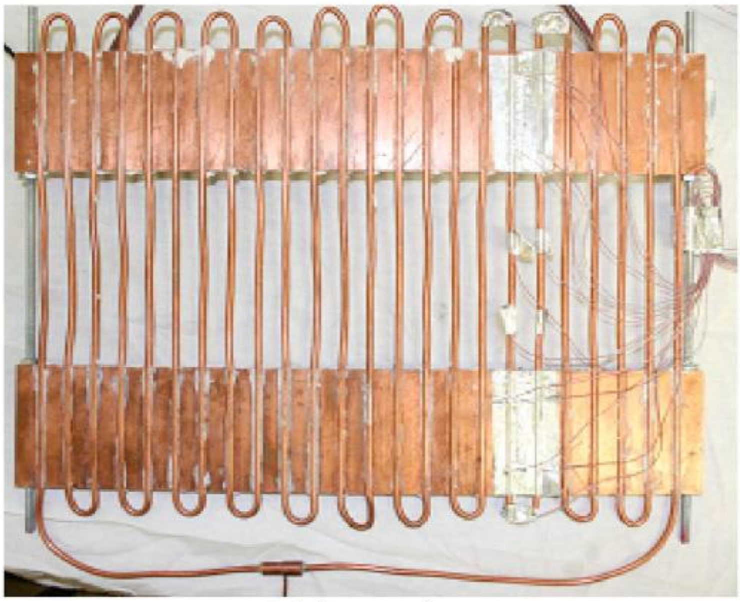


Figure8 *Nanofluid oscillating heat pipe*

I-6-3- Automotive Applications :

Automobile systems, including radiators, engines, heating systems, ventilation systems, and air conditioning (HVAC), have inherently poor heat transfer performance. These systems could benefit from the high thermal conductivity of nanofluids as a result of the addition of nanoparticles. Nanofluids can be applied in automobile applications–engine oils, automatic transmission fluids, coolants, lubricants, and other synthetic high temperature heat transfer fluids. We investigated one of these applications–nanofluidcoolant.

I-6-3-1- Nanofluidcoolant :

Nano-coolants are used to extract heat generated from cylinder heads, radiators, and automatic transmissions in automobiles. Generally, water and other synthetic liquids are used for this purpose. Nano-coolants, compared to water, have enhanced heat transfer properties. Also, less quantity and a lower flow rate are required, resulting in lower pumping power, lower initial cost of the equipment, and smaller size of the equipment[24].

I-6-4- Biomedical Applications:

In the biomedical industry, nanofluids and nanoparticles have wide applications including nanodrug delivery, cancer therapeutics, cryopreservation, nanocryosurgery, and sensing and imaging. We investigated nanodrug delivery.

I-6-4-1- Nanodrug Delivery:

Ghoshet et al[25]. reported that gold nanoparticles (AuNPs) provide non-toxic carriers for drug and gene delivery applications. With these systems, the gold core imparts stability to the assembly,whilehe monolayer allows tuning of surface properties such as charge and hydrophobicity. Figure 9 shows a schematic of a drug delivery system.

A drug delivery system using nanoparticles can be used to destroy cancerous cells by containing a carcinostatis substance in particles at the nanometer scale. Blood vessels, which supply nutrition, grow in a short time because cancerous cells grow faster than normal cells. Blood vessels located at cancerous cells are looser than those located at normal blood vessel. Therefore, the cancer will be able to be cured by the particles about 30nm which contain a carcinostatis substance.

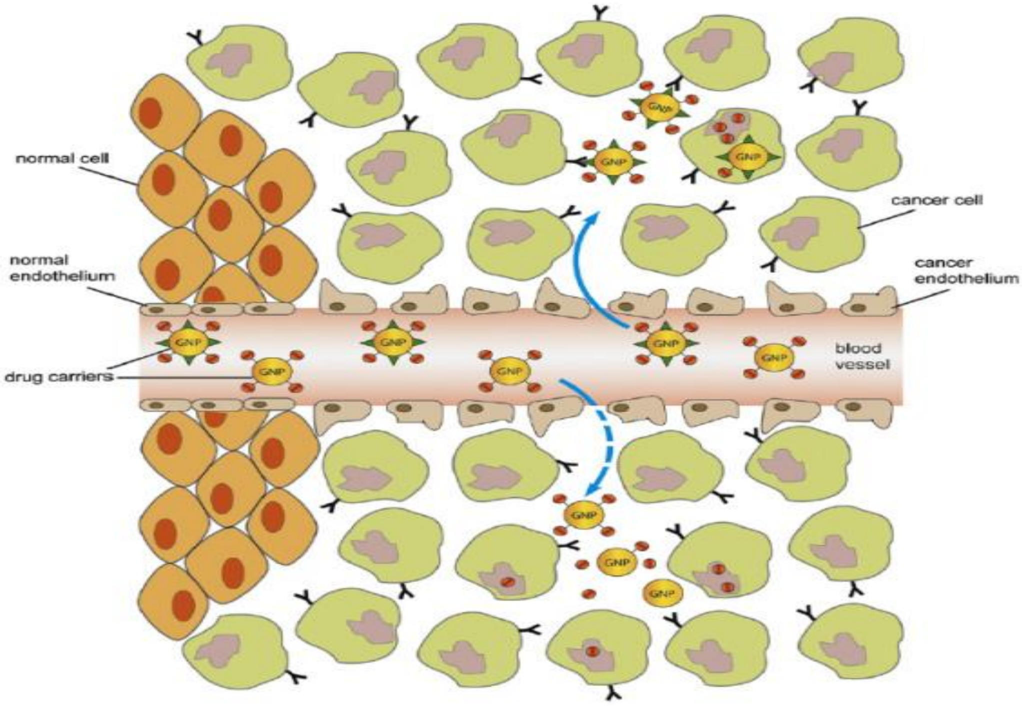


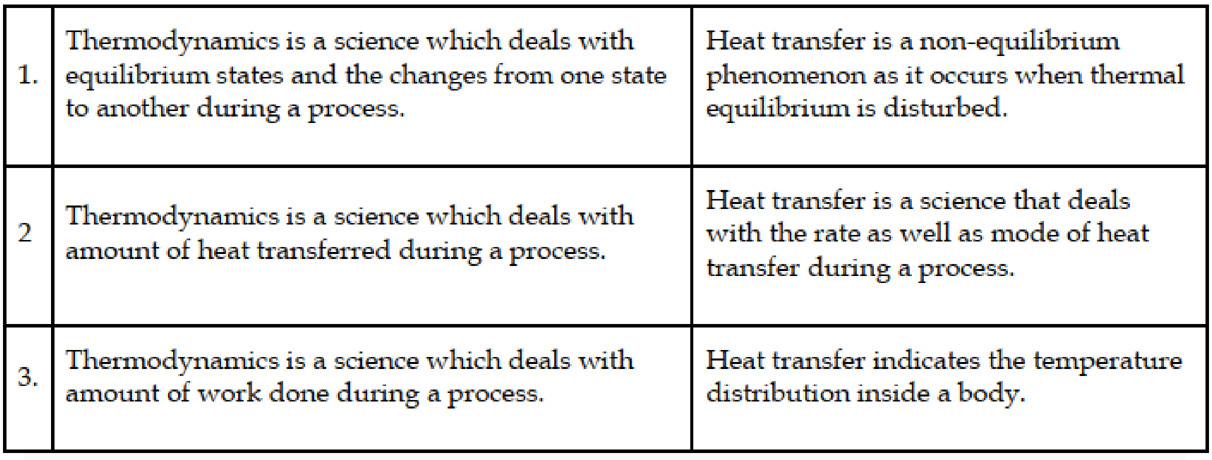
Figure9 Illustration of drug delivery via "active" and "passive" targeting (solid and dotted lines, respectively).

I-7- Nanofluids in heat transfer systems: pros and cons:

The heat transfer capacity of a nanofluid depends on several factors such as the concentration of the dispersed nanoparticles, the dispersion methodology, size, shape, and thermal conductivity of the nanoparticles, the base fluid type, nanofluid operating temperature, surfactant addition, and pH values, among others. The use of nanofluids in heat transfer systems has several pros and cons that should be considered. The main positive aspect of using nanofluids is that the addition of certain nanoparticles has the potential to enhance the thermal conductivity so they can increase the energy efficiency in different devices such as HPs . Nanoparticles have a larger surface area compared to micrometer-sized particles so, since heat transfer is a surface area-related property, it is possible to achieve better effects with a lower concentration. Furthermore, since boiling is the predominant heat transfer mechanism in HP, it is to be expected that by improving it the HP performance will increase. As shown in several studies, nanofluids improve boiling, which makes them a key element in improving HP performance. On the other hand, one of the main drawbacks of nanofluids is their stability. When a stable nanofluid is not achieved in the long term, problems of agglomeration and sedimentation of nanoparticles appear, and nanofluid thermal performance changes, decreasing device efficiency. Nanofluid stability can be improved by adding surfactants. However, these surfactants can decrease nanofluid thermal conductivity and increase its viscosity, negatively impacting the heat transfer. Therefore, stability is one of the main challenges that currently is focused on in many research efforts, so it is very important to know which factors are important to achieve a stable nanofluid.[13]

II-1- Heat Transfer and Thermodynamics:

Thermodynamics and heat transfer are related to each other. The laws of thermodynamicsform the basis of science of heat transfer. However, there are few fundamental differencesbetween thermodynamics and heat transfer which are given in Table 1:



1.	Thermodynamics is a science which deals with equilibrium states and the changes from one state to another during a process.	Heat transfer is a non-equilibrium phenomenon as it occurs when thermal equilibrium is disturbed.
2	Thermodynamics is a science which deals with amount of heat transferred during a process.	Heat transfer is a science that deals with the rate as well as mode of heat transfer during a process.
3.	Thermodynamics is a science which deals with amount of work done during a process.	Heat transfer indicates the temperature distribution inside a body.

Table 2 *: Fundamental Differences in Thermodynamics and Heat Transfer*

II-2- Relation of heat transfert to thermodynamics:

II-2-1- The first law with work equal to zero :

The supject of thermodynamics, as taught in engineering programs, makes constant reference to the heat transfert between systems the first law of thermodynamics for a closed system takes the following form on a rate basis:

$$Q=WK\ +\frac{dU}{dt}$$

When Q is the heat transfer rate and WK is the work transfer rate.they may be expressed in joules per second(j/s) or watts (w) .the derivate $\frac{dU}{dt}$ is the rate of change of internal thermal energy, U, with time,t.

II-3- Modes of Heat Transfer:

Heat transfer is the study of transmission of thermal energy from a high temperature region / body to a low temperature region / body on account of temperature difference. The rate of heat transfer is directly proportional to the temperature difference between the heat exchanging regions / bodies. Once the process of heat energy is complete, it is stored in one or more forms of energy such as potential, kinetic and internal energy. It is pertinent to mention that energy in transition as heat can never be measured; however, it is determined in terms of observed changes in other forms of energy. Transfer of heat between two regions / bodies maintained at different temperatures can occur in three different modes namely:

- Conduction
- Convection
- Radiation

In the conduction and convection modes, heat flows from high temperature to low temperature region / body whereas in radiation mode, transfer of heat takes place from both the bodies towards each other. However, net transfer of heat is always from high temperature body to low temperature body. Mechanism of heat transfer in each mode is different and controlled by different laws.[26]

II-3-1- Conduction:

Conduction refers to the transmission of energy from particles with higher energy levels to adjacent particles with lower energy levels through their interactions. This phenomenon occurs in solids, liquids, and gases.

Conduction involves the transfer of heat between regions of different temperatures within a body or between multiple bodies that are in direct physical contact. The process occurs through the exchange of molecular kinetic energy. According to the kinetic theory, the temperature of a body is directly proportional to the average kinetic energy of its constituent molecules. When the temperature in one region of a body increases, the kinetic energy of the molecules in that region also increases compared to the molecules in an adjacent region with a lower temperature.

In fluids, high-energy molecules transfer a portion of their energy to low-energy molecules through impacts, while in metals, this transfer occurs through diffusion. Consequently, the energy levels, and thus the temperature, of the low-energy molecules increase. This transfer of energy through molecular activity continues until the temperature becomes uniform along the entire length of the body. Please refer to Figure 10 for a visual representation of this process. [26]

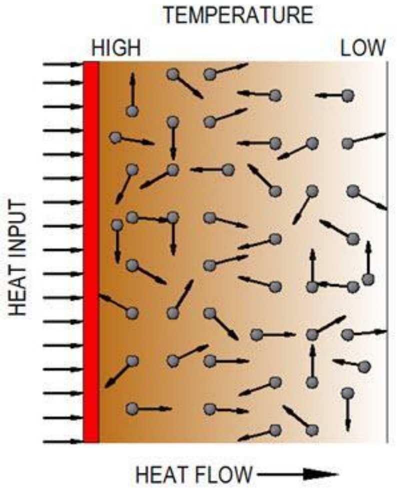


***Figure10** heat flow*

Heat transfer by conduction in solids, liquids and gases is determined by the thermal conductivity and temperature difference. The basic law of heat transfer by conduction was proposed by the French Scientist J. B. J. Fourier in 1822 and one dimensional Conduction rate equation described by the Fourier Law is written as:|

$$Q_X = -kA\frac{dT}{dx}$$

Where, Q_X - Heat Flow, (W)

k – Thermal conductivity of the material, (W/(m-K).
A – Cross-sectional area in the direction of heat flow, (m^2).
dT/dx – Temperature gradient, (K/m).[27]

II-3-1-1- Thermal Conductivity :

The thermal conductivity of a material refers to the rate at which heat is transferred through a unit thickness of the material per unit area, given a unit temperature difference. It is a measure of the material's ability to conduct heat. A high thermal conductivity value indicates that the material is an efficient heat conductor, while a low value suggests that the material is a poor conductor or acts as an insulator.

In heat transfer applications, the thermal conductivity of a fluid plays a vital role in determining its heat transfer capacity, both under still and flowing conditions. Enhancing the thermal capacity is desirable for equipment that requires a higher heat load. The size and efficiency of such equipment are directly influenced by their thermal capacity, and smaller sizes with higher efficiencies are preferred in various industrial applications. Adding nanoparticles to a base fluid generally leads to an improvement in thermal conductivity. It is commonly observed that the enhancement in thermal conductivity is more pronounced at elevated temperatures and for higher fractions of particles within the fluid.[13]

II-3-1-2- HEAT CONDUCTION EQUATION:

In this section we considered one-dimensional heat conduction and assumed heat conduction in other directions to be negligible. Most heat transfer problems encountered in practice can be approximated as being onedimensional, and we will mostly deal with such problems in this text. However, this is not always the case, and sometimes we need to consider heat transfer in other directions as well. In such cases heat conduction is said to be *multidimensional*, and in this section we will develop the governing differential equation in such systems in rectangular, cylindrical, and spherical coordinate systems.[28]

II-3-1-2-1- Rectangular Coordinates :

Consider a small rectangular element of length Δx, width Δy, and height Δz, as shown in Figure 11. Assume the density of the body is and the specific heat is *C*. An *energy balance* on this element during a small time interval Δtcan be expressed as

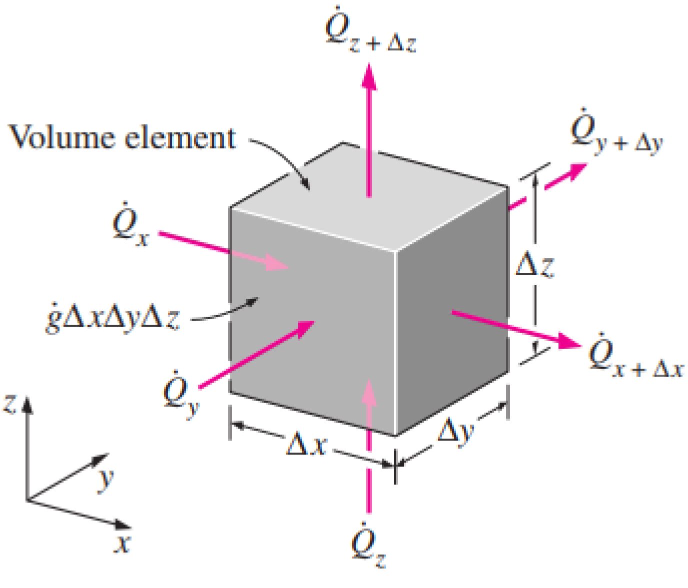


Figure11 *:Three-dimensional heat conduction through a rectangular volume element.*

$$\begin{pmatrix} rate\ of\ heat \\ conduction\ at \\ x, y\ and\ z \end{pmatrix} - \begin{pmatrix} rate\ of\ heat \\ conduction \\ at\ x+\Delta x, y+\Delta y \\ and\ z+\Delta z \end{pmatrix} + \begin{pmatrix} rate\ of\ heat \\ generation \\ inside\ the \\ element \end{pmatrix} = \begin{pmatrix} rate\ of\ change \\ of\ the\ energy \\ content\ of \\ the\ element \end{pmatrix}$$

Noting that the volume of the element is $v_{element} = \Delta x\Delta y\Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{element} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C\Delta x\Delta y\Delta z\,(T_{t+\Delta t} - T_t)$$

$$\dot{G}_{element} = \dot{g}v_{element} = \dot{g}\Delta x\Delta y\Delta z$$

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g}\Delta x\Delta y\Delta z = \rho C\Delta x\Delta y\Delta z\frac{(T_{t+\Delta t} - T_t)}{\Delta t}$$

Dividing by *x y z* gives

$$-\frac{1}{\Delta y\Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x\Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x\Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C\frac{(T_{t+\Delta t} - T_t)}{\Delta t}$$

Noting that the heat transfer areas of the element for heat conduction in the *x*, *y*, and *z* directions are $A_x = \Delta y\Delta z$, $A_y = \Delta x\Delta z$, $A_z = \Delta x\Delta y$, respectively, and taking the limit as $\Delta x, \Delta y, \Delta z$ and $\Delta t \rightarrow 0$ yields

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \quad (3)$$

In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

II-3-1-2-2- Cylindrical Coordinates :

The general heat conduction equation in cylindrical coordinates can be obtained from an energy balance on a volume element in cylindrical coordinates . It can also be obtained directly from Eq. 3 by coordinate transformation using the following relations between the coordinates of a point in rectangular and cylindrical coordinate systems:

$x = r\cos\emptyset$, $y = r\sin\emptyset$ and $z=z$

After lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{g}= \rho C\frac{\partial T}{\partial t}$$

II-3-1-2-3-Spherical Coordinates ;

The general heat conduction equations in spherical coordinates can be obtained from an energy balance on a volume element in spherical coordinates It can also be obtained directly from Eq3 by coordinate transformation using the following relations between the coordinates of a point in rectangular and spherical coordinate systems:

$x = r\cos\emptyset \sin\theta$, $y = r \sin\emptyset \sin\theta$ $z = \cos\theta$

Again after lengthy manipulations, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(k\, sin\theta\,\frac{\partial T}{\partial \theta}\right)+\dot{g}= \rho C\frac{\partial T}{\partial t}$$

Obtaining analytical solutions to these differential equations requires a knowledge of the solution techniques of partial differential equations, which is beyond the scope of this introductory text. Here we limit our consideration to one-dimensional steady-state cases or lumped systems, since they result in ordinary differential equations.[28]

II-3-1-3- BOUNDARY AND INITIAL CONDITIONS :

The heat conduction equations mentioned above were derived by considering an energy balance on a small element within the medium, and they remain unchanged regardless of the thermal conditions on the medium's surfaces. In other words, these differential equations do not incorporate any information about the surface conditions, such as the temperature or specified heat flux. However, we know that the heat flux and temperature distribution within a medium are influenced by the conditions at its surfaces. Therefore, describing a heat transfer problem in a medium requires a complete specification of the thermal conditions at the boundaries of the medium. These mathematical expressions that describe the thermal conditions at the boundaries are referred to as boundary conditions.

From a mathematical perspective, solving a differential equation involves eliminating derivatives or integrating the equation. Consequently, the solution of a differential equation generally involves arbitrary constants. To obtain a unique solution to a problem, it is necessary to provide more than just the governing differential equation. Additional conditions must be specified, such as the value of the function or its derivatives at certain points in the independent variable. By enforcing these conditions at specified locations, we can determine unique values for the arbitrary constants and obtain a unique solution. Since the differential equation itself does not provide a place to include this additional information or conditions, we need to supply them separately in the form of boundary or initial conditions.

II-3-2-Convection:

Convective heat transfer is a common mode observed in fluids. In reality, convection is a combination of two distinct mechanisms of energy transfer: heat diffusion (conduction) and advection (resulting from bulk fluid movement). Fluid motion occurs due to either a density gradient (natural buoyancy forces) or a pressure difference. When fluid motion is driven by density differences, it is known as "natural convection." On the other hand, when fluid flow is induced by a pressure gradient, it typically results from external surface forces generated through pumping, blowing, or other mechanically assisted methods. This is commonly referred to as "forced convection." It is worth noting that heat transfer in a fluid can involve a combination of natural and forced convection, known as "mixed convection." Forced convection is a mode of heat transfer that is encountered in numerous engineering processes, including heat exchangers, fan-assisted cooling, impinging jets, and more.

Convection is termed "forced convection" when external means such as fans, pumps, or wind are employed to force the fluid to flow over the surface. In contrast, convection is referred to as "natural convection" or "free convection" when fluid motion arises from buoyancy forces induced by density variations caused by temperature variations within the fluid. [28]

Heat transfer by convection occurs when a fluid (Liquid and gas) comes in contact with a solid through direct contact and a temperature difference exists between them. Heat transfer by Convection occurs under the combined action of heat conduction and mixing motion. When a fluid comes in contact with a hot surface, energy in form of heat flows by conduction from hot surface to the adjacent stagnant layer of fluid particles, thereby increasing their temperature and internal energy. Due to increase in temperature, density of the fluid particles decreases and they become lighter as compared to the surrounding fluid particles. The lighter fluid particles move up to a region of lower temperature with in the fluid where they mix and exchange a part of their energy with colder fluid particles. Simultaneously, the cold fluid particles move downwards to occupy the space vacated by hot fluid particles.This upward and downward movement of hot and cold fluid particles continues till temperature of the fluid and the surface becomes equal. The convection heat transfer process has been shown in Figure 12.[26]

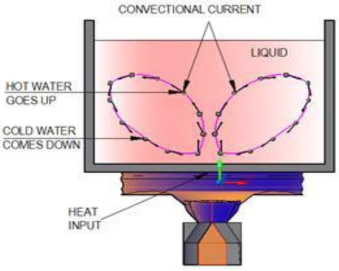


Figure12 The*convection heat transfer process*

Convective heat transfer rate is governed by Newton’s law of cooling and is expressed as

$$Q = hA_s(T_s - T_f)$$

Where, ‘h’ is convective heat transfer coefficient in W/ (m2 – K)
A_s is heat transferring area, m2

$T_s$andT_f are temperatures of surface and the fluid respectively, K[27]

II-3-2-1-Forced Convection Fundamentals :

II-3-2-1-1- Convection Heat transfer Coefficient :

Generally speaking, the rate of heat transfer by convection between a surface and a fluid can be calculated from Equation 5.1, initially proposed by Isaac Newton (often called Newton’s law of cooling):

$$\dot{Q}_C = \overline{h_c}A\left(T_W - T_{f,\infty}\right) = \overline{h_c}A\Delta T$$

where:
$\dot{Q}_C$ is the rate of heat transfer (W).
$\overline{h_c}$is the average convective heat transfer coefficient (W/m2K).
Ais the heat transfer surface area (m2).
T_W and$T_{f,\infty}$ are, respectively, the temperature at the surface of the solid and the fluid temperature at a reference location away from the surface (K).

II-3-2-1-2-Governing equations: Conservation of mass, momentum, and energy for laminar Flow :

To derive the conservation equations for laminar flow, one usually uses a control volume approach in which the rates of mass, momentum, and energy flowing into and out of the control volume are accounted for. The conservation of momentum arises from applying Newton's second law to fluid motion, whereas the conservation of energy equation represents the first law of thermodynamics. Details of the development of these equations can be found in any standard fluid mechanics textbook. Assuming that the fluid is incompressible and Newtonian and that both compression work and viscous dissipation are assumed negligible in the energy equation, the general conservation equations for steady flow can be written in vector form as follows (Warsi 1999):

- Conservation of mass :

$$\mathrm{Div}(\rho\vec{V}) = 0$$

- Conservation of momentum

$$\mathrm{Div}(\rho\vec{V}\vec{V}) = -\mathrm{grad}\, p + \mu\nabla^2\vec{V}$$

- Conservation of energy

$$\mathrm{Div}(\rho\vec{V}C_P \mathrm{T}) = \mathrm{div}(\mathrm{kgrad\ T})$$

where:
ρ, μ, C_P, and k are, respectively, the fluid density, viscosity, specific heat, and thermal conductivityof the considered fluid. T, p, and$\vec{V}$ are, respectively, the temperature, pressure and velocity vector.These equations are, except for highly simplified cases, impossible to solve analytically.[29]

II-3-2-2- dimensionless Groups of Importance in Forced Convection:

This section presents the most common dimensionless groups that are often encountered in forced convection applications such as thermal management systems.

- ***Reynolds number*** **(*Re*)**: The Reynolds number (Equation4) is based on fluid properties (density ρ and viscosity μ), a velocity "U_∞", and a characteristic length. It is the ratio of inertial forces to viscousforces for a given flow situation. As such, it represents the relative importance of these types of forces and is commonly used to characterize different flow regimes for fluid flows. For internal flows, the Reynolds number is typically based on the hydraulic diameter *D*h of the conduit and the mean flow velocity "U ".

$$Re = \frac{\rho U_\infty L}{\mu} = \frac{U_\infty L}{v_f}; \qquad \mathrm{Re} = \frac{\rho \bar{U} D_h}{\mu} \quad (4)$$

- ***Prandtl number* (*Pr*)**: The Prandtl number (Equation 5) represents the ratio of molecular momentum diffusivity (or kinematic viscosity ν) to thermal diffusivity α. Both are molecular transport properties. Essentially, the Prandtl number relates the temperature distribution to the velocity distribution. In practice, this means that for two geometrically similar configurations, similar temperature distributions will be found if both systems have the same Reynolds and Prandtl numbers.

$$Pr = \frac{C_p \mu}{k} = \frac{v}{\alpha} \quad (5)$$

***Nusselt number* (*Nu*):** The Nusselt number (Equation 6) is also known as the dimensionless heat transfer coefficient and is, as such, an essential parameter in forced convection applications. It represents the ratio of convection heat transfer to conduction in a fluid layer of thickness L. "*h*c " represents the average convective heat transfer coefficient and "*k*f" is the fluid thermal conductivity. In conduit flow, the characteristic length is typically the hydraulic diameter. In forced convection applications, the Nusselt number is usually expressed as a function of both Reynolds and Prandtl numbers, Nu = *f*(Re, Pr). A quick review of literature will reveal that empirical correlations of this type for a wide range of flows with heat transfer are available. One such example is the Dittus–Boelter correlation for pipe flow.

$$Nu = \frac{\overline{h_c L}}{k_f} \qquad (6)$$

- ***Peclet number* (*Pe*or *Pé*):** The Peclet number is the product of Reynolds and Prandtlnumbers (Pe = Re·Pr). By developing the Reynolds and Prandtl numbers in the Pecletnumber, one will see that it represents the ratio of the heat transport by convection to the heat transport by conduction.

- ***Friction factor* (*f*):** The friction factor (or *Darcy friction factor*) (Equation 7) represents the dimensionless pressure drop for internal flows through conduits. This coefficient is a function of factors such as the Reynolds number and, in the case of turbulent flow, the relative roughness of the pipe. In equation 7, Δp is the pressure drop, *L* and *D* are respectively the tube length and diameter, ρ is the fluid density and *U*m is the mean velocity.[29]

$$f = \frac{\Delta\rho}{(^{L}/_{D})(\rho U_m^2/2)} \quad (7)$$

II-3-2-3- Laminar and turbulent boundary layers:

II-3-2-3-1- Flow boundary layer:

Fluids flowing past solide bodies adhere to them, so a region of variable velocity must be built up between the body and the free fluid stream, asindicated in figure 13. this region is called a boundary layer , which we will often abbreviate has a thichnessδ . the boundary layer thickness is arbitrarly defined as the distance from the wall at whitch the flow velocity approaches to within 1% of u_∞ . the boundary layer is normaly very thin in comparison with the dimensions of the body immersed in the flow.[30]

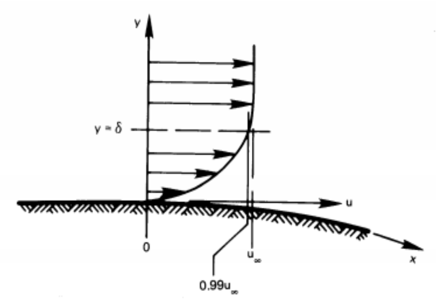


Figure13.*A boundary layer of thickness δ*

The first step that has to be taken before h can be predicted is the mathematical description of the boundary layer .this description was first made by prandlt and his student, starting in 1904; and it depended upon simplification that followed after he recognized how thin the layer must be.

The dimensional functionel equation for the boundary layer thickness on a flat surface is

$$\delta = fn(u_\infty, \rho, \mu, x)$$

Where x is the lenth along the surface and ρ and μ are the fluide density in kg/m^3 and the dynamic viscosity in $kg/m.s$.

II-3-2-3-2-Thertmal boundary layer:

If the wall is at a temperature T_w. Different from that of the free stream,T_∞ there is a thermal boundary layer thickness δ_t different from the flow thickness .a thermal is pictured in figure.14.now , with reference to this picture, we equate the heat conducted away from the wall by the fluid to the same heat transfer expressed in terms of a convective heat transfer coefficient:

$$-k_f \frac{\partial T}{\partial y}\bigg|_{y=0} = h(T_w - T_\infty)$$

Where k_f is the conductivity of the fluide .

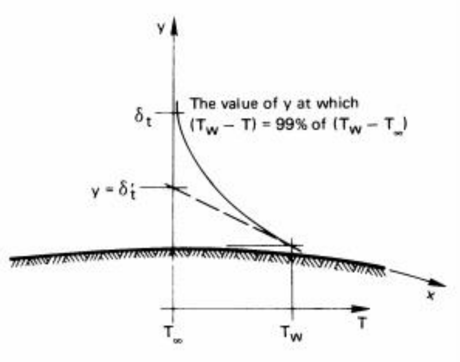


Figure14 *the thermal boundary layer during the flow of cool fluide over a warm plate*

II-3-2-4- Laminar incompressible boundary layer on a flat surface :

We predict the boundary layer flow field by solving the equations that express conservation of mass and momentum in the b.l. Thus, the first order of business is to develop these equations.

II-3-2-4-1- Conservation of mass - The continuity equation:

A two- or three-dimensional velocity field can be expressed in vectorial form:

$$u = \vec{\imath}u + \vec{\jmath}v + \vec{k}w$$

where*u*, *v*, and *w*are the *x*, *y*, and *z* components of velocity. Figure (15)shows a two-dimensional velocity flow field. If the flow is steady, thepaths of individual particles appear as steady *streamlines*. The streamlines can be expressed in terms of a *stream function*, *ψ(x, y)* = constant, where each value of the constant identifies a separate streamline,as shown in the figure.(15)

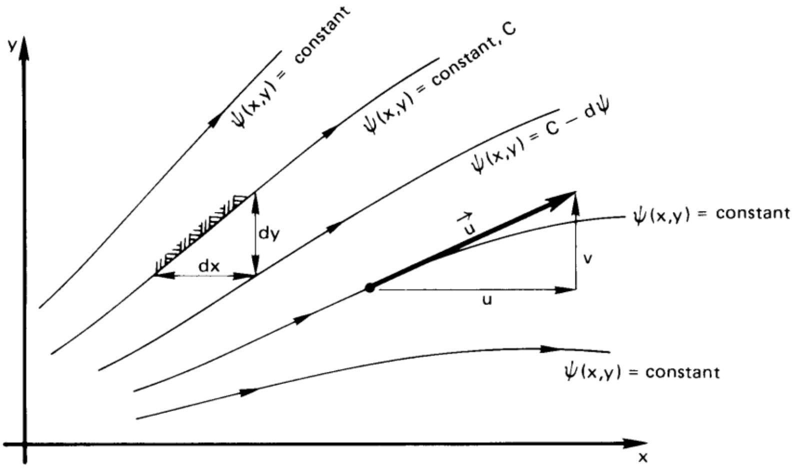


Figure15 *A steady, incompressible, two-dimensional flow field represented by streamlines, or lines of constant ψ.*

The equationis :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

This is called the two-dimensional *continuity equation* for incompressible flow, because it expresses mathematically the fact that the flow is *continuous*; it has no breaks in it. In three dimensions, the continuity equation for an incompressible fluid is

$$\nabla.\vec{u}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

II-3-2-4-2- Conservation of momentum :

The differential forms of the equations of motion in the velocity boundary layer are derived by applying Newton's second law of motion to a differential control volume element within the boundary layer. Newton's second law states that the net force acting on the control volume is equal to the product of mass and the acceleration of the fluid element within the control volume. This net force is also equivalent to the net rate of momentum outflow from the control volume.

The forces acting on the control volume can be categorized into two types. First, there are body forces that act throughout the entire volume of the control volume, such as gravity, electric forces, and magnetic forces. These body forces are proportional to the volume of the control volume. Second, there are surface forces that act on the control surface, including pressure forces due to hydrostatic pressure and shear stresses resulting from viscous effects. These surface forces are proportional to the surface area of the control volume.

During analysis, the control volume is isolated from its surroundings, and the effects of the detached body are replaced by a force located at that position. It is important to note that pressure represents the compressive force exerted by the surrounding fluid on the fluid element, and it always acts in the direction toward the surface.

In summary, by applying Newton's second law to the differential control volume within the boundary layer, the differential forms of the equations of motion can be obtained. These equations consider both the body forces acting throughout the volume and the surface forces acting on the control surface, including pressure forces and shear stresses due to viscosity.[28]

Conservation of Momentum Equation is :

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial^2 y}-\frac{\partial P}{\partial x}$$

This is the relation for the **conservation of momentum** in the *x*-direction, and is known as the ***x*-momentum equation.**

II-3-3- Radiation:

The transfer of heat between two bodies or mediums that are separated and not in direct contact with each other is known as radiation heat transfer. Unlike conduction and convection, radiation heat transfer does not rely on the presence of an intervening medium. It can occur through empty space or a vacuum.

Radiation heat transfer is based on electromagnetic waves, specifically infrared radiation, emitted by the surfaces of objects. These electromagnetic waves carry thermal energy and can travel through vacuum, air, or other transparent mediums. When the emitted radiation interacts with another object, it can be absorbed, transmitted, or reflected, resulting in the transfer of heat energy between the objects.

It is important to note that radiation heat transfer is independent of the presence of a material medium, allowing it to occur even in the absence of air or other substances. This property makes radiation heat transfer particularly effective in vacuum conditions, where conduction and convection are not possible due to the absence of molecules for direct energy transfer.(9)

‘radiarion’ is the transfert of heat through space or matter by means other there conduction or convection.

Radiation heat is thought of as electromagnetic waves or quanta (as convenient) an emanation of the same nature as light and radio waves. All bodies radiate heat; so a transfert of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits. Radiant energy (being electromagnetic) requires no medium for propagation and will pass through vaccum.[31]

ChapiterIII:Mathematical Formulation of the Problem and Numerical Resolution

III-1-Mathematical Formulation:

III-1-1-Studied configurations:

The studied configuration is represented in the figure.16. Consider steady two-dimensional stagnation point flow of optically dense tangent hyperbolic nanofluid towards a stretching cylinder located at $r= R$, where r is the coordinate normal to cylinder. The cylinder is stretched with two equivalent and conflicting forces along x-axis with the velocity$u=\frac{ax}{l}$, by keeping the origin fixed.

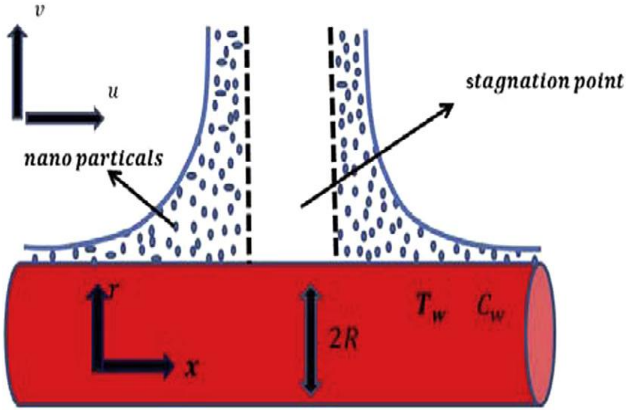


Figure16 *Geometry of the problem.*

It is assumed that concentration and temperature at the wall is maintained at constant concentration C_w and temperature T_w. Where C1 and T1 are ambient concentration and temperature respectively.

III-1-2-Simplifying assumptions :

The simplifying assumptions adopted in our study are as follows:

●the developed model of a tangent hyperbolic nanofluid in boundary layer flow with Brownian motion and thermophoresis effects are discussed.

●The nanofluid is assumed to be incompressible, and the flow is assumed to be laminar, unsteady, and fully developed in two dimensions.

III-1-3-equations of the mass, momentum, energy and nanoparticles in cylindrical coordinates:

The deviation in viscosity and thermal conductivity with temperature are supposed to be in the form

$$\mu \ = \ \mu_0 e - \zeta^{(T - T\infty)},$$

$$k^* = k(1 + \epsilon\theta)$$

the governing equations in cylindrical coordinates x and r are given by :

$$\frac{\partial(ru)}{\partial \mathrm{x}}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial r} = \left(v\,(1-n)\,\frac{\partial^2 u}{\partial x^2} + (1-n)\,\frac{1}{r}\frac{\partial u}{\partial r} + n\sqrt{2}T\,\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r^2} + \frac{\partial T}{\partial X}\left(\frac{\partial y}{\partial x}\right)^2\right) + U_e\,\frac{dU_e}{dx} \tag{2}$$

$$\rho c_p\,(u\frac{\partial T}{\partial X} + v\,\frac{\partial T}{\partial r}) = \frac{K}{r}\frac{\partial}{\partial r}\,(r\frac{\partial T}{\partial r}) + \tau\left\{D_B(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}) + \frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial r}\right)^2\right]\right\} + \frac{Q}{Pc_p}\,(T - T_\infty) \tag{3}$$

$$u\frac{\partial\,c}{\partial X} + v\,\frac{\partial c}{\partial r} = \frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) + \frac{D_T}{T_\infty} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{4}$$

Here u and v are the velocity components along x and r directions respectively, Q is the temperature dependent volumetric rate of heat source when $Q > 0$ and heat sink when $Q < 0$, dealing with the situation of exothermic and endothermic chemical reactions respectively, m is the kinematic viscosity, q is the density, A is the stretching ratio parameter, Γ is the Williamson parameter, n the power law index, DB is the Brownian diffusion coefficient, $C_P$is the specific heat, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$is theratio between the effective heat capacity of the nanoparticles material and heat capacity of the fluid, $D_T$is the thermophoresis diffusion coefficient and C is resealed nanoparticles volume fraction.

III-1-4-the associated boundary conditions :

at$r = R$: $U = \frac{ax}{l} v = 0\,,\ T = T_w\,,\ c = c_w$

at$r \rightarrow \infty$: $U = U_e(x) = \frac{bx}{l}\ ,\ v = 0\ ,\ T = T_\infty\ ,\ C = C_\infty$ (5)

Using the similarity transformations :

$$\eta = \sqrt{\frac{a}{lv}}\,(\frac{r^2 - R^2}{2R})\psi = \sqrt{\frac{vc}{l}}\,xRf(\eta)$$

$$\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}h(\eta) = \frac{c - c_\infty}{c_w - c_\infty} \tag{6}$$

$$u = \frac{1}{r}\frac{\partial\psi}{\partial r}v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$$

Eqs: (1) - (5) takes the form

$$(1-n)(1+2K\eta)f''' + ff''' - (f') + 2\mathrm{K}(1-n)f'' + 2\lambda n(1+2K\eta)^{\frac{3}{2}}f''f''' + 3\lambda k(1+2K\eta)^{\frac{1}{2}}(f'')^2 + A^2 = 0 \quad (7)$$

$$(1+2K\eta)\theta'' + 2\mathrm{K}\,\theta' + \mathrm{Pr}(\theta' f + (1+2K\eta)\theta' h' N_b + N_t + (1+2K\eta)\theta'^2) + Pr\beta\theta = 0 \qquad (8)$$

$$(1+2K\eta)h'' + 2\mathrm{k}h' + Pr\mathrm{L}efh' + \frac{N_t}{N_b}(1+2K\eta)\theta'' + 2\mathrm{k}\frac{N_t}{N_b}\theta' = 0 \qquad (9)$$

$$f(0) = 0\,, \quad f'(0) = 1\,, \quad \theta(0) = 1\,, \quad h(0) = 1$$

$$f'(\infty) = \mathrm{A}\,, \theta(\infty) = 1\,, \quad h(\infty) = 0 \qquad (10)$$

wherek; b; N_b; N_t; Le; A and Pr denotes curvature parameter dimensionless Weissenberg number, heat generation/absorption Brownian motion parameter, thermophoresis parameter, Lewis number, stretching ratio parameter and Prandtl number which are given by:

$$P_r = \frac{v}{\alpha}\lambda = \frac{a}{\frac{3}{l2}}\sqrt{\frac{a}{2v}}\,\Gamma x$$

$$K = \frac{1}{R}\sqrt{\frac{vl}{a}}\,A = \frac{b}{a} \qquad (11)$$

$$N_b = \frac{\tau D_B(C_w - C_\infty)}{\rho c_p v} N_t = \frac{\tau D_t(T_w - T_\infty)}{\rho c_p v T_\infty}$$

$$Le = \frac{\alpha}{D_B}\beta = \frac{Q}{\rho c_p a}$$

Skin friction coefficient Cf :

$$c_f = \frac{\tau_w}{\frac{\rho a^2 x^2}{2l^2}}\tau_w = \mu\left((1-\mathrm{n})\frac{\partial \mathrm{u}}{\partial \mathrm{r}} + \frac{n\Gamma}{\sqrt{2}}\left(\frac{\partial \mathrm{u}}{\partial \mathrm{r}}\right)^2\right)_{r=R}$$

localNusselt number Nux :

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)}q_w = -\mathrm{k}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{r}}\right)_{r=R}$$

local Sherwood number Shx :

$$sh_x = \frac{x h_m}{k(C_w - C_\infty)} h_m = -\mathrm{k}\left(\frac{\partial \mathrm{C}}{\partial \mathrm{r}}\right)_{r=R}$$

while the dimensionless forms of skin friction coefficient, local Nusselt number and Sherwood number are:

$$\frac{C_f {Re_x}^{1/2}}{2} = (1-\mathrm{n})\mathrm{f}''(0) + \mathrm{n}\lambda \mathrm{f}''^{2}(0) \frac{Nu_x}{{Re_x}^{1/2}} = -\theta'(0) \frac{Sh_x}{{Re_x}^{1/2}} = -h'(0)$$

Where: $Re_x = \mathrm{a}^{1/2}\, x / v^{1/2}\, l^{1/2}$

III-2-Numerical Resolution:

III-2-1-shooting method :

The shooting method is a numerical approach for solving boundary value problems by converting them into initial value problems. It involves "shooting" a trajectory from one boundary towards the other and adjusting the initial conditions iteratively until the desired boundary conditions are met.

In more detail, the shooting method works as follows:

1. The boundary value problem is transformed into an initial value problem by introducing an unknown initial condition.
2. An initial guess for this unknown initial condition is made.
3. An ordinary differential equation solver is used to integrate the system from one boundary towards the other using the guessed initial condition.
4. The resulting solution is checked against the desired boundary conditions.
5. If the boundary conditions are not met, the initial guess for the unknown initial condition is adjusted, and the integration is repeated.
6. This process is repeated iteratively until the solution converges to satisfy the boundary conditions within an acceptable tolerance.

The shooting method is widely used in various fields, including physics, engineering, and numerical analysis, to solve boundary value problems that cannot be solved directly using standard techniques like finite differences or finite elements. It's particularly valuable for problems where the boundary conditions are not specified at both ends of the domain or when other methods may be computationally expensive or impractical.

III-2-2-Newton–Raphsonmethod:

The Newton–Raphsonmethodis an iterativemathematical technique used to approximate the roots (or solutions) of a real-valuedfunction f(x). It startswith an initial guess for the root and refinesthisguessthrough successive iterations, using the function'sderivative to converge rapidlytoward the trueroot.

The methodrepeatsthisprocessuntil a satisfactory approximation to the rootisachieved, oftenwithinaspecifiedtolerance or after a fixednumber of iterations.

Numerical Resolution:

The nonlinearcoupledordinarydifferentialequations (7)-(9) are solved by usingshootingmethod. The step size $\Delta\eta = 0.1$ is chosen to obtain numerical solution. As the energy, momentum and concentration equations are of order second, third and second respectively. After converting Eqs. (7)-(9) into first-orderequationstakes the form

$$y_1' = y_2 \tag{12}$$

$$y_2' = y_3 \tag{13}$$

$$y_3' = \frac{\left((y_2)^2 - y_1 y_3 - 2k(1-n)y_3 - 3k\lambda(1+2k\eta)^{1/2} - A^2\right)}{(1-n)(1+2k\eta) + 2n\lambda(1+2k\eta)^{3/2} y_3} \tag{14}$$

$$y_4' = y_5 \tag{15}$$

$$y_5' = \frac{\left(-2ky_4 - Pr(y_1 y_5 - N_b(1+2k\eta)y_5 y_7 + N_t(1+2k\eta)(y_5)^2\right) - Pr\beta y}{(1+2k\eta)} \tag{16}$$

$$y_6' = y_7 \tag{17}$$

$$y_7' = \frac{\left(-2Ky_7 - PrLe\; y_1 y_7 - \frac{N_t}{N_b}(1+2k\eta)y_5' - 2k\frac{N_t}{N_b}y_5\right)}{(1+2k\eta)} \tag{18}$$

The boundary conditions become

$y_1 = 0$,$y_2 = 1$, $y_4 = 1$, $y_6 = 1$ at $\eta = 0$

$y_2 \to A, y_4 \to 0$, $y_6 \to 0$ as $\eta \to \infty$ (19)

Here four conditions are known and three of them are unknown. Donating the unknown initial conditions byU_1; $U_2$and U_3.

$y_1 = 0$,$y_2 = 1$,$y_3 = U_1$,$y_4 = 1, y_5 = U_1$, 6=1 ,$y_7 = U_3$, $\eta = 0$ (20)

Eqs. (12)-(18) are solvedwith the help of initial conditions defined in Eq. (20). The computed boundary values at $\eta = 4$ depends on thechoice of U_1; $U_2$and U_3. The accurate choice ofU_1; U_2 and U_3delivers the givenboundary conditions at$\eta = 4$ that is it satisfies the Eq.(21) (boundaryresidual)

$$\emptyset_1(U_1, U_2, U_3)=0.1 \quad , \quad \emptyset_2\,(U_1, U_2, U_3)=0 \quad , \quad \emptyset_2(U_1, U_2, U_3)=0 \tag{21}$$

If U_1; U_2 and U_3does not satisfy the boundaryresidualthentheir values willberefined by using Newton–Raphsonmethod.

IV-Résultats et discussion :

IV-1-Introduction:

Shooting method is used to solve the nonlinear ordinary differential equations (7)-(9) with boundary conditions Eq. (10) and thebehavior of curvature parameter K, stretching ratio parameter A,Brownian motion parameter N_b, thermophoresis parameter N_t,heat generation/absorption β, Prandtl number Pr, power law indexn, Weissenberg number λ and Lewis number Le on velocity, temperature and concentration profiles are illustrated through graphs.Weused the Maple software in ourstudy by entering the equations and boundary values.

We arewriting the calculation program in the maplelanguage ,then the curvesweregeneratedusingOrigin 2019b software.

In order to check the accuracy of the solution, the values of $f''(0)$ are calculated for unlike values of stretching ratio parameter A,by ignoring the effects of Lewis number, curvature parameter,thermophoresis parameter, dimensionless Weissenberg number,Brownian motion parameter and heat generation/absorption. The results are compared with the values calculated by Mahapatra[32] and Ibrahim [33] as shown in Table 3. Table 4 shows the com-parison of local Nusselt number -$\theta'(0)$ for different values of Prandtl number Pr, calculated by Mahapatra [32] and Ibrahim [33]. These tables show that the solution obtained by present paper and the solution obtained by Mahapatra [32] and Ibrahim [33] are approximately equal.

Table 3 :

Comparison of $f''(0)$with the previousexistingliteraturewhenn=0 $\lambda = 0$; $\beta = 0$;$N_t = 0$; $N_b \to 0$; Le = 0; $\lambda = 0$ and K = 0.

A	Mahapatra	Ibrahim	Presentresults
0.01	-0.9980	-0.9980	-0.9980
0.1	-0.9694	-0.9694	-0.9694
0.2	-0.9181	-0.9181	-0.9181
0.5	-0.6673	-0.6673	-0.6673
2	2.0175	2.0175	2.0175
3	4.7292	4.7292	4.7292

Table 4:

Comparison of results local Nusseltnumber-$\theta'(0)$ for several values of Pr and A when n=0 $\lambda = 0$; $\beta = 0$;$N_t = 0$; $N_b \to 0$; Le = 0; $\lambda = 0$ and K = 0.

Pr	A	Mahapatra	Ibrahim	Presentresults
1	0.1	0.603	0.6022	0.6022
	0.2	0.625	0.6245	0.6247
	0.5	0.692	0.6924	0.6927
1.5	0.1	0.777	0.7768	0.7776
	0.2	0.797	0.7971	0.7975
	0.5	0.8648	0.8648	0.8648

Tables 5 - 7 shows the behavior of the skin friction coefficient, local Nusseltnumber and local Sherwood numberrespectively for different values of parameters. It isobservedthat local Nusseltnumberisdecreasingfunctionwhile local Sherwood numberis an increasingfunction for unlike values of dimensionlessparametersN_b ,N_t, Le and Pr.

Table 5 :

Values of skin friction coefficient$\frac{C_f\sqrt{Re_x}}{2}$with respect to A,λ, n and K.

A	K	λ	n	$(1-n)f''(0)+n\lambda f''^2(0)$
0.1	0.1	0.1	0.1	-0.9546
0.2				-0.9020
0.3				-0.8333
0.1	0.1			-0.9546
	0.2			-0.9894
	0.3			-1.0243
	0.1	0.1		-0.9546
		0.2		-0.9540
		0.3		-0.9535
		0.1	0.1	-0.9546
			0.2	-0.8967
			0.3	-0.8355

Table 6 :

Values of local Nusseltnumber$\frac{Nu_x}{{Re_x}^{1/2}}$with respect to N_b ,N_t,β and Pr.

N_b	N_t	Pr	β	-$\theta'(0)$
0.1	0.1	1.2	0.1	0.5565
0.2				0.5205

0.3				0.4858
0.1	0.1			0.5565
	0.2			0.5296
	0.3			0.5034
	0.1	1.2		0.5565
		1.3		0.5745
		1.4		0.5921
			0.1	0.5565
			0.2	0.4467
			0.3	0.3172

Table 7 :

Values of local Sherwood number$\frac{Sh_x}{{Re_x}^{1/2}}$ with respect to N_b ,N_t, Le and Pr.

N_b	Le	N_t	Pr	$-h'(0)$
0.1	1.2	0.1	1.2	0.2514
0.2				0.3943
0.3				0.4415
0.1	1.2			0.2514
	1.3			0.2980
	1.4			0.3436
	1.2		1.2	0.2514
			1.3	0.2784
			1.4	0.3058

Fig.17 presents the behavior of curvatureparameter K on velocity profile. It is evident from figure that velocity increases with increasing values of curvature parameter K. Near the surface of cylinder velocity reduces due to friction of the wall to the fluid par-ticles. The augmentation in velocity is due to the fact that after increasing curvature parameter K, radius of curvature decreases which causes area of the cylinder to reduce. Hence less resistance is offered by the cylinder to the fluid, so velocity increases.

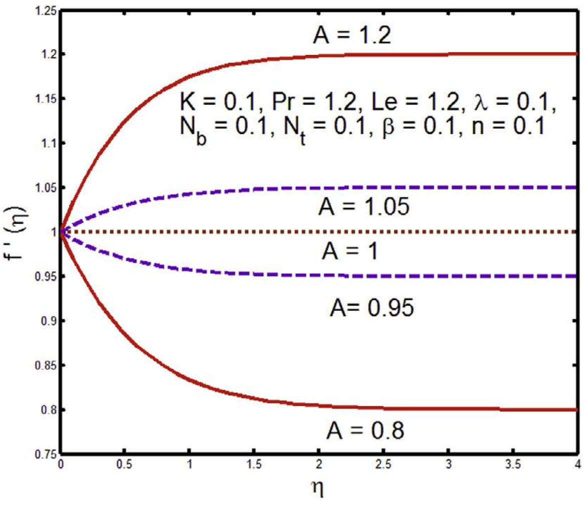


***Fig17**. Influence of A on* $f'(\eta)$.

Figure.18. shows how stretching ratio parameter A affects the velocity profile.It is noticed that when free streamvelocityexceeds the stretching velocity of the cylinder, the fluidvelocityincreases and boundarylayer thicknessreduceswithincrease in stretching ratio parameter A. Moreover, when free streamvelocityislessthan the stretching velocity of the cylinder, the fluidvelocitydecreases and boundary layer thicknessincreasesmonotonically.

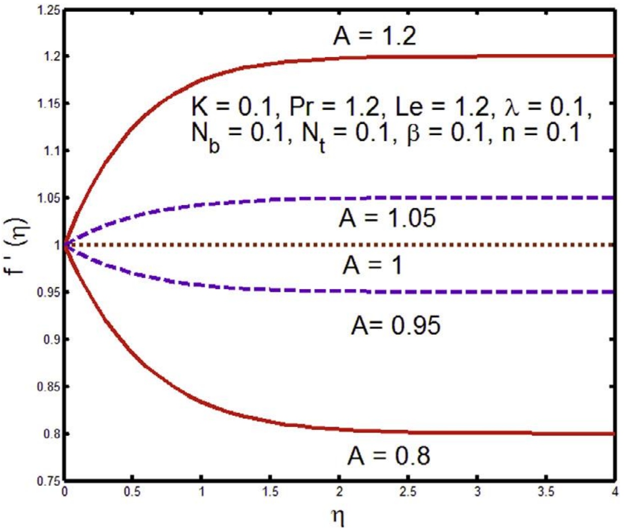


***Fig. 18.** Influence of A on* $f'(\eta)$.

Figure.19.depicts the influ-ence of Weissenbergnumberλ on velocity profile. It isobservedfrom figure, reduction in velocityisnoticed for eachincrement in value of Weissenbergnumberλ, because after increasing Weis-senberg number λ the relaxation time increaseswhichoffers more resistance to flow and hencevelocityreduces.

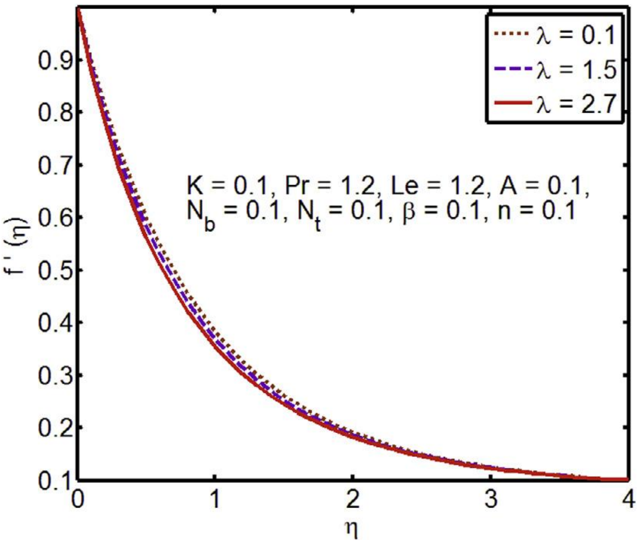


Fig. 19. *Influence of* λ *on* $f^{'}(\eta)$.

Concludingremarks :

The effects of nanofluid and stagnation point flow over a stretching cylinder are investigatednumerically. Tangent hyperbolicfluidisconsidered as a base fluid. In order to check the endothermic and exothermicchemicalreactions, heatgeneration/absorption effectisalsoincluded. A well-known technique (shootingmethod) isused to calculate the comparison, graphical and tabularresults for the governing system of equations. In the light of presentanalysisfollowingdeductionsmaybedrawn.

- ✓ A qualitativelydifferentbehaviorwasseen in the velocitypro-file for different values of stretching ratio parameter (A > 1 andA < 1).
- ✓ Thickening of the thermal boundary layer wasobserved for higher values of heatgeneration/absorption β.
- ✓ The thermal boundarylayerthicknessreduces for increasing values of prandtlnumber Pr.
- ✓ Influence of thermophoresisparameterN_t on nanoparticles concentration issimilar as compared to the temperature.
- ✓ Velocity profile reduces for increasingvalues of weissenbergnumber λ and power law index n.

References :

[1] Efstathios E. Michaelides, Transport properties of nanofluids.A critical review. De Gruyter, 2013.

[2] MOHAMMAD HATAMI DENGWEI JING, NANOFLUIDS Mathematical, Numerical, and Experimental Analysis. Elsevier 2020.

[3] Eastman, J., et al., 2001, “Anomalously Increased Effective Thermal Conductivities of Ethylene-Glycol-Based Nanofluids Containing Copper Nanoparticles,” Appl. Phys. Lett., 786, pp. 718–720.

[4] Das, S., et al., 2003, “Temperature Dependence of Thermal Conductivity Enhancement for Nanofluids,” J. Heat Transfer, 125, pp. 567–574.

[5] Pak, B. C., and Cho, Y., 1998, “Hydrodynamic and Heat Transfer Study ofDispersed Fluids With Submicron Metallic Oxide Particles,” Exp. Heat Transfer, 11, pp. 151–170

[6] Brinkman, H. C., 1952, “The Viscosity of Concentrated Suspensions and Solutions,” J. Chem. Phys., 20, pp. 571–581.

[7] Batchelor, G. K., 1977, “The Effect of Brownian Motion on the Bulk Stress ina Suspension of Spherical Particles,” J. Fluid Mech., 831, pp. 97–117.

[8] Maïga, S., et al., 2004, “Heat Transfer Behaviors of Nanofluids in a Uniformly Heated Tube,” SuperlatticesMicrostruct., 35, pp. 543–557.

[9] Lee, S., et al., 1999, “Measuring Thermal Conductivity of Fluids Containing Oxide Nanoparticles,” J. Heat Transfer, 121, pp. 280–289.

[10] Wang, X., et al., 1999, “Thermal Conductivity of Nanoparticle-Fluid Mixture,”J. Thermophys. Heat Transfer, 134, pp. 474–480

[11] Xuan, Y., and Li, Q., 2003, “Investigation on Convective Heat Transfer and Flow Features of Nanofluids,” J. Heat Transfer, 125, pp. 151–155.

[12] Sarit K. Das Stephen U. S. Choi Wenhua Yu T. Pradeep NANOFLUIDS Science and Technology A JOHN WILEY & SONS, INC., PUBLICATION 2008.

[13] Hafiz Muhammad Ali, Advances in Nanofluid Heat Transfer, Elsevier , 2022.

[14] K.R.V. Subramanian TubatiNageswaraRaoAvinashBalakrishnanNanofluids and Their Engineering Applications taylor and francis 2020.

[15] Yong-Han Yoon JaeSool Shim A Review on Nano Applications: Fluids and HeatTransfer Journal of the Korean Society for Power System Engineering, Vol. 17, No. 5, pp. 3-12, October 2013.

[16] Argonne National Laboratory, 2009, "Development, Characterization, Production, and Demonstration ofNanofluids for Industrial Cooling Applications.“

[17] K. Ma and J. Liu, 2007, "Nano liquid-metal fluid as ultimate coolant.",Phys. Lett. A, Vol. 361, pp. 252–256.

[18] G. Huminic and A. Huminic, 2012, "Application of nanofluids in heat exchangers: A review.", Renewable and Sustainable Energy Reviews, Vol. 16, pp. 5625–5638

[19] B. Chun, H. U. Kang and S. H. Kim, 2008, "Effect of alumina nanoparticles in the fluid on heat transfer in double-pipe heat exchanger system", Korean J. Chem. Eng., Vol. 25, No.5, pp. 966-971.

[20] F. Khoddamrezaee, R. Motallebzadeh, D. JajaliVahid, 2010, "Simulation of (EG+Al2O3) nanofluid through the shell and tube heat exchanger with rectangular arrangement of tubes and constant heat flux", Journal of Applied Scizzences, Vol. 10, Issue 6, pp.500– 505.

[21] Y. J. Hwang, Y. C. Ahn, H. S. Shin, C. G. Lee, G. T. Kim, H. S. Park and J. K.Lee,2005, "Investigation on characteristics of thermal conductivity enhancement ofnanofluids.", Current Applied Physics, Vol. 6, Issue 6, pp. 1068-1071.

[22] Z. H. Han, F. Y. Cao and B. Yang,2008, "Synthesis and thermal characterization of phase-changeable indium/polyalphaolefinnanofluids.", Appl. Phys. Lett., Vol. 92, pp. 243104-243106.

[23] H. B. Ma, C. Wilson, B. Borgmeyer, K. Park, Q. Yu, S. U. S. Choi and M. Tirumala, 2006, "Effect of nanofluid on the heat transport capability in an oscillating heat pipe", Appl. Phys. Lett., Vol. 88, No. 14, pp. 143116~143118.

[24] K. V. Sharma, K. Kadirgama, W. A. W. Hamzah, R. Mamat, R. A. Bakar, R. Prasad and A. Gupta, 2011, "NANOFLUIDS –THE FUTURE INDUSTRIAL COOLANTS."

[25] P. Ghosh, G. Han, M. De, C. K. Kim and V. M. Rotello, 2008, "Gold nanoparticles in delivery applications.", Advanced Drug Delivery Reviews, Vol. 60, pp. 1307–1315.

[26] yunus A.CENGEL AFSHIN J.GHAJAR HEAT AND MASS TRANSFERFUNDAMENTALS & APPLICATIONS MC GRAW HILL.2015

[27] VishvjeetHans Dr. Varun, Heat and Mass Transfer ,agrimoon.com.

[28] yunus A.CENGEL, HEAT TRANSFER A PQRTICAL APPROACH SECOND EDITION, GRAW HILL.

[29] Vincenzo Bianco, OronzioManca ,Sergio Nardini ,KambizVafai ,Heat TransferEnhancement with Nanofluids ,CRC PRESS ,2015.

[30] johnH.lienhard IV and john H. lienhard V , a heat transfer textbook third edition, phlogiston press ,2008.

[31] Er.RK.RAJPUT, heat and mass transfer, s.chand and company LTD.2011.

[32] Dessie H, Kishan N. MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. Ain Shams Eng J 2014;5:967–77.

[33] Hakeem AKA, Kalaivanan R, Ganesh NV, Ganga B. Effect of partial slip on hydromagnetic flow over a porous stretching sheetwith non-uniformheat source/sink, thermal radiation and wall mass transfer. Ain Shams Eng J 2014;5:913–22.